Distributions with locally free tangent sheaf

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In 2008 F. Cukierman and J. V. Pereira published a study of the locus Dec, where the tangent sheaf of a family of foliations in $\mathbf{P}^n$ is decomposable, i.e. a sum of line bundles. They concluded that, once the singular locus has sufficiently large codimension, Dec turns out to be open. I shall report on work done with J. V. Pereira on the theme. In it, we study the locus LF of points of a family of distributions where the tangent sheaf is locally free. Through general Commutative Algebra, we show that LF is open provided that singularities have codimension at least three. This, in turn, is applied to families in $\mathbf{P}^n$ and in $\mathcal{B}$, the variety of Borel subgroups of a simple group. With the help of a theorem putting in bijection irreducible components of the space of subalgebras of a given semi-simple Lie algebra and its nilpotent orbits, we conclude that the space of foliations of rank two, on $\mathbf{P}^n$ and $\mathcal{B}$, may have quite many irreducible components.