

The Calabi problem for Fano 3-folds

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The Calabi problem asks which compact complex manifolds admit a Kähler-Einstein metric. A necessary condition for the existence of a Kähler-Einstein metric is that the canonical class of the manifold has a definite sign. For manifolds with zero or positive canonical class, the Calabi problem was solved by Yau and Aubin/Yau in the 1970s. They confirmed Calabi's prediction, showing that these manifolds always admit a Kähler-Einstein metric. On the other hand, projective manifolds with negative canonical class, called "Fano manifolds", may or may not admit a Kähler-Einstein metric. The Yau-Tian-Donaldson conjecture, which is now a theorem, states that a Fano manifold admits a Kähler-Einstein metric if and only if it satisfies a sophisticated algebro-geometric condition, called "K-polystability". In the last few years, tools from birational geometry have been used with great success to investigate K-polystability. In this talk, I will discuss the Calabi problem, its connections with birational geometry, and the current state of the art in dimension 3.