Transversely Projective Structures on Smooth Foliations on Surfaces

Given a regular holomorphic foliation of codimension one on a complex manifold, we may wonder what happens when we assume certain properties. Between these cases, we have foliations that admits a holomorphic transverse projective structure. This means that the foliation is defined by submersions to the complex projective line and that the transition between them can be done by a Möbius transformation. When we are on a compact surface, we can use the classification of regular foliations on compact complex surfaces made by M. Brunella in 1997 to see that all foliations have a meromorphic projective transversal structure, but not necessarily holomorphic. A careful look at Brunella's list just cited reveals that besides fibrations, turbulent foliation, and foliations on Hopf surfaces, every other smooth foliation on a compact complex surface admit a natural transversely projective structure which is holomorphic. This structure tells us how to make local transportations of the leaves of the foliation.

The results obtained in collaboration with Jorge Vitório Pereira and Gabriel Fazoli shows which foliations admits projective transversal structure and which do not, by using Brunella's classification of regular foliations and analyzing the monodromy representation of the transversal structure. The goal of this talk is to present the case where the ambient space is a compact Kähler surface.