

FOLIATIONS AND THE GEOMETRY OF COMPLEX ELLIPTIC SURFACES

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We will discuss two foliations that turn out to be useful in studying the geometry of complex (Jacobian) elliptic surfaces.

First, every family of elliptic surfaces carries a canonical codimension two foliation which turns out to be integrable in the sense that its leaves are the level sets of algebraic functions. This foliation plays an important role in the proof that a very general elliptic surface of Kodaira dimension 1 contains only finitely many rational curves.

Second, any elliptic surface carries a canonical codimension 1 foliation which is in general not integrable or algebraic. This non-holomorphic object can be used to bound tangencies between multiples of a section and the zero section thereby addressing a question of “unlikely intersections.”