Tropical semirings and curve singularities

Ethan Cotterill (Unicamp)

Abstract: Curve singularities are classical objects of study in algebraic geometry. The key player in their combinatorial structure is the value semigroup, or its compactification, the value semiring. One natural problem is to explicitly determine the value semirings of distinguished infinite classes of singularities, with a view to understanding their asymptotic properties. In this paper, we establish a matroidal framework for systematically resolving this problem. More precisely, we show how to associate to any curve singularity a support semiring that maps homomorphically to the value semiring. This is a tropical semiring with a finitary matroid structure, and we show how its basic features explain well-known features of value semirings of singularities, including a natural characterization of minimal generating sets. In the case of either line arrangements (i.e., multiple points) or cusps, we can be more quantitatively precise; and our results have important consequences for the topology of Severi varieties of singular rational curves in projective space.