

# Hypertoric varieties, $W$ -Hilbert schemes and Coulomb branches

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Motivated by physics, in the late 1990s Sen discussed a construction of complete hyperkähler metrics in (real) dimension 4 and so-called ALF (asymptotically locally flat) asymptotics as a "superposition" of simpler explicit building blocks, namely  $\mathbb{Z}_2$ -invariant multi-Taub-NUT metrics and the Atiyah-Hitchin metric. These metrics were then produced by Cherkis–Kapustin and Cherkis–Hitchin, amongst others, via twistor theory and Nahm's equations. In this talk I will discuss joint work with R. Bielawski about a higher dimensional version of this story. We study transverse equivariant Hilbert schemes of hypertoric varieties invariant under the action of a Weyl group  $W$ . We investigate the conjectural hyperkähler metric on these spaces in terms of twistor theory and Nahm's equations and discuss the relation of (symplectic quotients of) such Hilbert schemes with the Coulomb branches of 3-dimensional  $N=4$  supersymmetric gauge theories in theoretical physics, recently defined as holomorphic symplectic varieties by Braverman–Finkelberg–Nakajima.