Cubic Differentials and Harmonic Maps into an Asymptotic Cone

John Loftin¹

¹ Rutgers University

Higgs bundles, due originally to Hitchin, provide a way to produce many equivariant harmonic maps from the universal cover of a Riemann surface into symmetric spaces of noncompact type. In general, the construction involves non-explicit solutions to systems of elliptic PDEs (the Hitchin equations) and ODEs (a parallel transport). We are able to say more in an important special case.

Consider a Riemann surface S of genus g at least 2 equipped with a holomorphic cubic differential U. Following Hitchin, Labourie and myself, we construct a Higgs bundle from U over S so that the Hitchin system reduces to a single PDE, and the induced harmonic map is a minimal embedding into the symmetric space X=SL(3,R)/SO(3). Along a real ray tU as t goes to infinity, we find an explicit description of the geometry of the limiting minimal surface in the asymptotic cone of X, in terms of the geometry of U.

This is joint work with Andrea Tamburelli and Mike Wolf.