Complex curves in hypercomplex nilmanifolds

Iuliia Gorginian¹ ¹ IMPA

A nilmanifold is a compact manifold obtained as a quotient of a nilpotent Lie group by a cocompact lattice. When a Lie group is equipped with a left-invariant complex structure, the resulting nilmanifold is called a complex nilmanifold. It is known that almost all complex nilmanifolds, except a torus, are non-Kahler. Among all complex non-Kahler surfaces only three types of Inoue surfaces have no curves. It is natural to study the existence of complex curves in a non-Kahler manifold. For example, any deformation of the Iwasawa manifold, which is a 3-dimensional complex nilmanifold, possesses a complex curve. We prove that a general twistor deformation of a hypercomplex nilmanifold admits no complex curves.