Rigidity of Einstein metrics - the case of compact symmetric spaces

Paul Schwahn

Université Paris Saclay

The deformation theory of Einstein metrics on compact manifolds is tricky. Infinitesimal deformations in the moduli space of Einstein metrics may be characterized by a certain eigenspace of the Lichnerowicz Laplacian on trace- and divergence-free symmetric 2-tensors. But these deformations are in general obstructed - in fact one expects countably many polynomial obstructions against integrability. The problem of rigidity, i.e. whether there exist unobstructed infinitesimal Einstein deformations (IED), has been studied on homogeneous Einstein manifolds to some extent. Recently, there has been a number of rigidity results for some of the irreducible compact Riemannian symmetric spaces which possess IED (Batat-Hall-Murphy-Waldron 2021, Nagy-Semmelmann 2023). We consider a Hermitian symmetric example, the family of complex Grassmannians, and show that on half of them, all IED are obstructed to second order. Furthermore, we explore the challenges that come with working out the higher order obstructions.