Title: Holomorphic conformal structures on the moduli space of complex structures of lattice polarized K3 surfaces

Abstract: Period domains of complex algebraic varieties often enjoy rich geometric structure, and such is certainly the case for algebraic K3 surfaces equipped with a lattice polarization, or equivalently, equipped with an arrangement of algebraic and transcendental curves on the surface. Following Dolgachev, a lattice polarized K3 surface is the natural setting for K3 surface mirror symmetry, and it follows from the associated local and global Torelli theorems that the period domain lies on a quadric hypersurface in projective space as a quasi-projective subvariety. As complex quadric hypersurfaces are homogeneous spaces for the complex conformal group, and hence serve as the model space of holomorphic conformal geometry, it follows that the complex structure moduli space for such a lattice polarized K3 naturally inherits a holomorphic conformal structure that is locally conformally flat. Results of Sasaki & Yoshida imply that such a geometric structure is both necessary and sufficient to completely characterize the local variation of complex structure of the K3 surface, via the associated Picard-Fuchs equations annihilating the period integrals of the K3. While these results have been known for some time, they do not appear to have garnered much subsequent traction. We apply their framework to compute the Picard-Fuchs equations of generic K3 Kummer surfaces, obtaining a novel and compact description of the local variation of complex structure. In turn, we compute an explicit locally conformally flat holomorphic metric on the relevant complex structure moduli space. Based on submitted work with Andreas Malmendier: https://arxiv.org/abs/2401.10950