Killing vector fields on compact pseudo-Kähler manifolds are holomorphic

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It is well-known that Killing vector fields on compact Kähler manifolds are necessarily real-holomorphic. It was claimed by Yamada in 2012 that the same conclusion was true for compact pseudo-Kähler manifolds as well (that is, without assuming that the metric is positive-definite), but there was a gap in his proof. We explain this gap and offer two different proofs of the claimed result under the additional assumption that the underlying complex manifold has the $\partial\overline{\partial}$ property. Lastly, in real dimension 4, the $\partial\overline{\partial}$ property turns out to be unnecessary. We do not know if the $\partial\overline{\partial}$ assumption can be removed in higher dimensions. This is joint work with Andrzej Derdzinski, `https://arxiv.org/abs/2309.09820`.