Isospectral CR manifolds with respect to the Kohn Laplacian

In Spectral Geometry, it is important the Inverse Problem, that is, how much of the geometric information of a manifold is encode in the Spectrum of the Laplace Operator. This was popularized with the famous question Can one hear the shape of a drum? One of the goals of the area is to find isospectral examples, that is, objects with the same spectra but with different shape.

The Kohn Laplacian has been deeply studied, in many articles, and also its spectrum on Cauchy-Riemann manifolds (CR manifolds, for short). Isospectrality with respect to this operator has been firstly explored recently by Fan, Kim, Plzak, Shors, Sottile and Zeytuncu [JGA, 2023]. They dealt with lens spaces endowed with the CR structure induced by the canonical CR structure on the sphere, where they proved a rigidity result in 3 dimensional lens spaces. The question whether there are examples of CR isospectral manifolds remained open. In this work we construct examples of CR isospectral lens spaces, which are not CR diffeomorphic. We also show, for each odd dimension, an infinite family of pairs of such lens spaces.