

Interest in Riemannian manifolds with holonomy equal to the exceptional Lie group  $G_2$  have spurred extensive research in geometric flows of  $G_2$ -structures defined on seven-dimensional manifolds in recent years. Among many possible geometric flows, the so-called isometric flow has the distinctive feature of preserving the underlying metric induced by that  $G_2$ -structure, so it can be used to evolve a  $G_2$ -structure to one with the smallest possible torsion in a given metric class. This flow is built upon the divergence of the full torsion tensor of the flowing  $G_2$ -structures in such a way that its critical points are precisely  $G_2$ -structures with divergence-free torsion. In this article we study three large families of pairwise non-equivalent non-closed left-invariant  $G_2$ -structures defined on simply connected solvable Lie groups previously studied in the literature and compute the divergence of their full torsion tensor, obtaining that it is identically zero in all cases.