ON THE CANONICAL BUNDLE OF THE COMPLEX MANIFOLDS UNDERLYING A COMPACT HYPERCOMPLEX MANIFOLD

We provide the first example of an 8-dimensional compact hypercomplex manifold $(M, \{J_1, J_2, J_3\})$ such that the complex manifold (M, J_1) has (holomorphically) trivial canonical bundle, while the complex manifolds (M, J_2) and (M, J_3) have non trivial canonical bundle. As a consequence, the holonomy of the Obata connection associated to $(M, \{J_1, J_2, J_3\})$ is not contained in $\mathrm{SL}(2, \mathbb{H})$. The manifold M is a solvmanifold, that is, a quotient of a simply connected solvable Lie group G by a co-compact discrete subgroup (i.e., a lattice), and the hypercomplex structure is invariant, that is, it is induced by a left invariant hypercomplex structure on G.

In order to show the results stated above, we study the canonical bundle of solvmanifolds equipped with invariant complex structures. We characterize the complex solvmanifolds that admit an invariant trivializing section of the canonical bundle, and we exhibit examples where such a section is not invariant. We also provide a necessary condition for the canonical bundle to be trivial (in fact, holomorphically torsion).

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