Rational homotopy type of closed 7- and 8-manifolds

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Deligne, Griffiths, Morgan and Sullivan proved that any closed Kähler manifold is formal in the sense of rational homotopy theory; thus their the rational homotopy type is completely determined by the cohomology algebra. However, it remains an open problem whether closed 7-manifolds with holonomy G_2 and closed 8-manifolds with holonomy Spin(7) must be formal. The simplest obstructions to formality are non-zero Massey products in the cohomology, but because the known constructions of G_2 and Spin(7)-manifolds are complicated, even evaluating the Massey products of such examples is involved.

Defining tensors on the cohomology of a space by multiplying triple or fourfold Massey products by a further cohomology class gives an object that has less dependence on choices than the Massey products themselves, making them easier to work with. On the other hand, for a closed oriented manifold, Poincare duality allows all Massey products to be recovered from these tensors. Moreover, suitable interpretations of these tensors can capture information about the rational homotopy type even when the Massey products are undefined, and they have nice functoriality properties. For closed k-connected manifolds of dimension up to 5k+2 (k 1), these tensors (along with the cohomology algebra itself) suffice to determine the rational homotopy type. Conjecturally the same is true up to dimension 6k+2, but at least the vanishing of the tensors is equivalent to formality in that case. This is based on joint work with Diarmuid Crowley and Csaba Nagy.