

*Surfaces with Wahl singularities and their many applications*

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**Abstract:** Wahl singularities are the two-dimensional quotient singularities that admit a smoothing with Milnor number equal to zero. Originally, these singularities appeared in the work of Wahl in the early 80ies. Soon after, they were very relevant in the 1988 work of Kollár and Shepherd-Barron on deformations of quotient singularities and construction of a compactification of the moduli space of surfaces of general type, and later, from another point of view, in the 1997 rational blowdown surgery for 4-manifolds by Fintushel, Stern, and Park. The global set-up is a projective surface with only Wahl singularities together with a one-parameter smoothing, which is locally the Milnor number zero smoothing at each singularity. We abbreviate this situation by the name of *W-surface*. It turns out that W-surfaces have a rich theory analogous to the classical theory of nonsingular projective surfaces: intersection relative to the base, explicit birational maps to minimal models (MMP for W-surfaces), and explicit canonical models defining families in the Kollár–Shepherd-Barron–Alexeev compactification. They have many applications: construction of exotic 4-manifolds (and the relation to the geography of rational configurations), geography of surfaces of general type, optimal boundedness of singularities in W-surfaces of general type, constructions of semi-orthogonal decompositions of the derived category of the varieties involved, etc. We plan to briefly introduce the W-surface theory, including the explicit MMP, and to show some of these applications with the aim of describing current open questions.