Low-action holomorphic curves and invariant sets

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In this talk, I'll report on some joint work in progress with Dan Cristofaro-Gardiner. We use holomorphic curves to search for nontrivial invariant sets of i) rational area-preserving diffeomorphisms of closed surfaces and ii) Reeb flows of torsion contact forms on closed 3-manifolds. The former class of examples contains Hamiltonian surface diffeomorphisms and the latter class of examples contains Hamiltonian flows on star-shaped energy levels in R^4. For any system in i) or ii), we show that a) the system is not minimal (i.e. has a non-dense orbit) on the complement of any proper compact invariant set and b) the closure of the union of periodic orbits is not an isolated invariant set. We do not assume that the system has finitely many periodic orbits. This is a generalization, in the smooth conservative setting, of results of Le Calvez-Yoccoz and Franks for homeomorphisms of the two-sphere. The main tools are feral curves and PFH/ECH.