Components of Hodge loci of minimal codimension

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The Noether-Lefschetz locus is the space of surfaces of degree d in P^3 with Picard number greater than 1. One of the central questions answered by Voisin and Green was to show that the smallest codimension component of the Noether-Lefschetz locus corresponds to the locus of surfaces containing a line.

The analog in higher dimension of the Noether-Lefschetz locus is the so-called Hodge locus, which corresponds to the space of projective hypersurfaces P^{n+1} of even dimension n and degree d that contains at least one non-trivial Hodge cycle. Otwinowska showed that the smallest codimension component of the Hodge locus of hypersurfaces corresponds to the locus of hypersurfaces containing a linear subvariety of dimension $\frac{n}{2}$ for $d >> n$. To obtain these results; Voisin, Green and Otwinowska found an upper bound for the dimension of the Zariski tangent space of all local Hodge loci V_λ, associated with a Hodge cycle λ, parameterizing some local component of the Hodge locus. **These, together with other evidence, led Movasati to conjecture that the only local Hodge loci V_λ around the Fermat variety whose tangent space reaches the minimum codimension are the local Hodge loci V_λ with λ a linear cycle.** In this talk, we will reveal and give a complete answer to this conjecture.