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Local central limit theorem for triangle counts in sparse random graphs

Let X_3 be the number of triangles in $G(n, p)$, with mean μ and variance σ. Here we prove a local central limit theorem for X_3 whenever $Cn^{-1/2} \leq p \leq 1/2$ and $C > 0$ is large enough. More precisely, defining $X_3^* = (X_3 - \mu)/\sigma$, we prove in this regime of p that

$$\sup_{x \in \mathcal{L}_{X_3}} \left| \frac{1}{\sqrt{2\pi}} e^{-x^2/2} - \sigma \cdot \mathbb{P}(X_3^* = x) \right| \to 0,$$

where $\mathcal{L}_{X_3} := \frac{1}{\sigma}(\mathbb{Z} - \mu)$ is the support of X_3^*. Our proof relies on the connection between point probabilities and the characteristic function of a random variable.