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Slow graph bootstrap percolation

Given a fixed graph H and an n-vertex graph G the *H-bootstrap percolation process* of H on G is defined to be the sequence of graphs G_i, $i \geq 0$ which starts with $G_0 := G$ and in which G_{i+1} is obtained from G_i by adding every edge that completes a copy of H. This process is an example of a cellular automaton and has been extensively studied since being introduced by Bollobás in 1968. Recently, Bollobás raised the question of determining the maximum running time of this process, over all choices of n-vertex graph G. Here, the running time of the process is the number of steps t the process takes before stabilising, that is, when $G_t = G_{t+1}$. Recent papers of Bollobás–Przykucki–Riordan–Sahasrabudhe, Matzke and Balogh–Kronenberg–Pokrovskiy–Szabó have addressed the case when H is a clique, and determined the asymptotics of this maximum running time for all cliques apart from K_5. Here, we initiate the study of the maximum running time for other graphs H and provide a survey of our new results in this direction. We study several key examples, giving precise results for trees and cycles, and giving general results towards understanding how the maximum running time of the H-bootstrap percolation process depends on properties of H, in particular exploring the relationship between this graph parameter and the degree sequence of H. Many interesting questions remain and along the way, we indicate some directions for future research.

This represents joint work with David Fabian and Tibor Szabó.