Large values of the error term in the prime number theorem

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Assume the Riemann hypothesis throughout. A classic result of von Koch gives an upper bound for the error term in the prime number theorem which is sharp up to logarithms. Determining the largest fluctuations of this error term is an open problem with the only progress conditional on Montgomery's pair correlation conjecture. In this talk I'll introduce some techniques which give new estimates for the measure of the set of large values. The basic idea is to show large values of the error term in the prime number theorem correspond to Bohr sets containing a little more zeros of the Riemann zeta function than expected. Many such Bohr sets allow one to run a density increment strategy to obtain one Bohr set with much more zeros than expected. This is enough to obtain a contradiction from Montgomery's work on the average density of zeros in short intervals.