Free Boundary Problems in PDEs and Related Issues

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Título: A singular perturbation problem for the normalized $p(x)$-Laplacian operator.

Resumo: In the work we focus our attention in looking for a family of solutions $(u_\varepsilon)_{\varepsilon>0}$ – in the viscosity sense – for the singularly perturbed problem driven by the normalized $p(x)$-Laplacian operator

$$\left\{ \begin{array}{rcll} \Delta^{\mathrm{N}}_{p_\varepsilon(x)} u_\varepsilon(x) & = & \zeta_\varepsilon(u_\varepsilon) + f_\varepsilon(x) & \text{in} \quad \Omega, \\ u_\varepsilon(x) & = & g(x) & \text{on} \quad \partial\Omega, \end{array} \right.$$

where p_ε, ζ_ε and f_ε are suitable continuous date. In this context, we show that such solutions satisfy certain analytical and geometric properties, namely, they enjoy uniform boundedness (vi **A.B.P** estimate), they are Lipschitz continuous and fulfill a non-degeneracy property, for a smooth bounded domain $\Omega \subset \mathbb{R}^N$ and a regularly boundary datum g. As a result, we show that, up to subsequence $\lim\limits_{j\to\infty} u_{\varepsilon_j}(x) = u_0(x)$, with $u_0 \in C^{0,1}(\Omega; [0, L])$ satisfying, in the viscosity sense, a one-phase free boundary problem of Bernoulli type as follows

$$\left\{ \begin{array}{rcll} \Delta^{\mathrm{N}}_{p_0(x)} u_0(x) & = & f_0(x) & \text{in} \quad \{u_0 > 0\}, \\ u_0(x) & = & g(x) & \text{on} \quad \partial\Omega, \\ u_0(x) & \geq 0, & & \end{array} \right.$$

where $p_0(x) = \lim\limits_{\varepsilon\to 0^+} p_\varepsilon(\mathrm{x})$ and $f_0(x) = \lim\limits_{\varepsilon\to 0^+} f_\varepsilon(x)$. Such a class of free boundary problem must be understand as the non-variational counterpart of certain problems in flame propagation coming from combustion theory. This is a joint work with J.V. da Silva (UNICAMP - Brasil).