A singular variant of the Falconer distance problem

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In this talk we will discuss the following variant of the Falconer distance problem. Let E be a compact subset of $\mathbb{R}^d$, $d \geq 1$, and define

$$\square(E) = \{|(y,z) - (x,x)| : x, y, z \in E,\ y \neq z\} \subseteq \mathbb{R}.$$

We showed using a variety of methods that if the Hausdorff dimension of E is greater than $\frac{d}{2} + \frac{1}{4}$, then the Lebesgue measure of $\square(E)$ is positive. This problem can be viewed as a singular variant of the classical Falconer distance problem because considering the diagonal (x,x) in the definition of $\square(E)$ poses interesting complications stemming from the fact that the set $\{(x,x) : x \in E\} \subseteq \mathbb{R}^{2d}$ is much smaller than the sets for which the Falconer type results are typically established.