Free Boundary Problems in PDEs and Related Issues

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Título: Optimal regularity to the obstacle problem for p-Laplacian type equations.

Resumo: In this Lecture, we will present sharp gradient estimates for weak solutions of an obstacle problem for a second-order quasilinear operator with degenerate/singular signature of p-Laplacian type and (possibly) unbounded source terms, namely, we prove $C^{1,\alpha}_{\text{loc}}$ for an explicit regularity exponent $\alpha = \alpha(n, p, q, \beta) > 0$ depending only on universal parameter and data. These results are striking even for the simplest model case governed by the p-Laplacian operator

$$\begin{cases} \Delta_p u & = & f(x) & \text{in} & \{u > \varphi\} \cap B_1 \\ \Delta_p u & \leq & f(x) & \text{in} & B_1 \\ u(x) & \geq & \varphi(x) & \text{in} & B_1 \\ u(x) & = & 0 & \text{on} & \partial B_1, \end{cases}$$

where $f \in L^q(\Omega)$ with $q > \dfrac{np}{p-1}$ (for $1 < p < \infty$) and $\varphi \in C^{1,\beta}(\Omega)$. This is a joint work with J.V. da Silva (UNICAMP - Brasil).