34° Colóquio Brasileiro de Matemática

Algebraic curves with a transitive action on the Weierstrass points

Speaker: Pietro Speziali (IMECC-UNICAMP)
Joint work with: Sebastián Reyes-Carocca

Let X be an algebraic curve of genus $g \geq 2$ defined over an algebraically closed field of characteristic $p \geq 0$. We say that X is classical with respect to the canonical series if the gap-sequence at a generic point P of X is equal to $\{1, \ldots, g\}$, non-classical otherwise. If $p = 0$, any curve is classical, while if $p > 0$, this is no longer true, with many well-known examples.

In any of the cases above, a classical result states that X has a finite number of Weierstrass points, that is, points whose gap-sequence is different from the generic one. Next, assume that X admits a non-trivial group of automorphisms G; in this context, G is always finite.

Since G must act on the set W of Weierstrass points of X, it is natural to ask if this action can be transitive. As it turns out, this is a rare possibility, with very few known examples in the literature, and until recently, only one infinite family of examples, namely the famous Accola-Maclachlan curve for $p = 0$.

In this talk, we give an overview of the literature on this subject and our contributions. More in detail, we prove that, if $p = 0$ of $p \geq 5$, there exists an infinite family of pairwise non-isomorphic classical curves of genus 3 admitting a transitive action o W. Finally, we discuss some further developments of this work.