

# **$r$ -PRIMITIVE $k$ -NORMAL ELEMENTS IN ARITHMETIC PROGRESSIONS OVER FINITE FIELDS**

JOSIMAR J.R. AGUIRRE  
UNIVERSIDADE FEDERAL DE UBERLÂNDIA  
MG - BRASIL

ABSTRACT. Let  $\mathbb{F}_{q^n}$  be a finite field with  $q^n$  elements. For a positive divisor  $r$  of  $q^n - 1$ , the element  $\alpha \in \mathbb{F}_{q^n}^*$  is called  *$r$ -primitive* if its multiplicative order is  $(q^n - 1)/r$ . Also, for a non-negative integer  $k$ , the element  $\alpha \in \mathbb{F}_{q^n}$  is  *$k$ -normal* over  $\mathbb{F}_q$  if  $\gcd(\alpha x^{n-1} + \alpha^q x^{n-2} + \dots + \alpha^{q^{n-2}} x + \alpha^{q^{n-1}}, x^n - 1)$  in  $\mathbb{F}_{q^n}[x]$  has degree  $k$ . In this talk we discuss the existence of elements in arithmetic progressions  $\{\alpha, \alpha + \beta, \alpha + 2\beta, \dots, \alpha + (m-1)\beta\} \subset \mathbb{F}_{q^n}$  with  $\alpha + (i-1)\beta$  being  $r_i$ -primitive and at least one of the elements in the arithmetic progression being  $k$ -normal over  $\mathbb{F}_q$ . We obtain asymptotic results for general  $k, r_1, \dots, r_m$  and concrete results when  $k = r_i = 2$  for  $i \in \{1, \dots, m\}$ .