

LOWER BOUNDS FOR THE ORDER OF ELEMENTS IN FINITE FIELD EXTENSIONS

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ABSTRACT. For many important applications (for example AKS algorithm), it is interesting to find an element of very high order in a finite extension field \mathbb{F}_{q^n} . Ideally, one would choose a primitive element, but actually finding such an element is a notoriously hard computation problem. Many authors have worked, in order to show an element for which a reasonably large lower bound on the order can be guaranteed.

In this work, we find a lower bound for the order of the group $\langle \theta + \alpha \rangle \subset \overline{\mathbb{F}}_q^*$, where $\alpha \in \mathbb{F}_q$, θ is a generic root of the polynomial $F_{A,r}(X) = bX^{q^r+1} - aX^{q^r} + dX - c \in \mathbb{F}_q[X]$ and $ad - bc \neq 0$. In addition, we find a lower bound for the order of a generic element of $\mathbb{F}_q(\theta)$ of the form $\theta^e(\theta^f + a)$, where $a \in \mathbb{F}_q^*$ and $-1 \in \langle q \rangle \subset \mathbb{Z}_{\text{ord}(\theta)}^*$.