

Energy Estimates and Multiplicity of Seminodal Solutions to an Elliptic System with Mixed Couplings

Mayra Soares¹, Mónica Clapp²

¹ Universidade de Brasília

² Universidad Nacional Autónoma de México

We study the system of semilinear elliptic equations

$$-\Delta u_i + u_i = \sum_{j=1}^{\ell} \beta_{ij} |u_j|^p |u_i|^{p-2} u_i, \quad u_i \in H^1(\mathbb{R}^N), \quad i = 1, \dots, \ell,$$

where $N \geq 4$, $1 < p < \frac{N}{N-2}$, and the matrix (β_{ij}) is symmetric and admits a block decomposition such that the entries within each block are positive or zero and all other entries are negative.

We provide simple conditions on (β_{ij}) , which guarantee the existence of fully nontrivial solutions, i.e., solutions all of whose components are nontrivial.

We establish existence of fully nontrivial solutions to the system having a prescribed combination of positive and nonradial sign-changing components, and we give an upper bound for their energy when the system has at most two blocks.

We derive the existence of solutions with positive and nonradial sign-changing components to the system of singularly perturbed elliptic equations

$$-\varepsilon^2 \Delta u_i + u_i = \sum_{j=1}^{\ell} \beta_{ij} |u_j|^p |u_i|^{p-2} u_i, \quad u_i \in H_0^1(B_1(0)), \quad i = 1, \dots, \ell,$$

in the unit ball, exhibiting two different kinds of asymptotic behavior: solutions whose components decouple as $\varepsilon \rightarrow 0$, and solutions whose components remain coupled all the way up to their limit.

KEYWORDS: Nonlinear elliptic system, weakly coupled, mixed cooperation and competition, positive and sign-changing components, singularly perturbed elliptic system.