TWO-DIMENSIONAL JORDAN ALGEBRAS: CLASSIFICATION AND POLYNOMIAL IDENTITIES THEORY

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Abstract. Jordan algebras were introduced by the physicist Pascual Jordan to formalize notions in quantum mechanics. These algebras have been studied since then and have connections with other areas of Mathematics such as Differential Geometry, Functional Analysis and Projective Geometry. Precisely, a Jordan algebra J is a commutative algebra whose product satisfies

$$((x^2)y)x - (x^2)(yx) = 0,$$

for all x, $y \in J$.

Let F be a field of characteristic different from 2. Small-dimensional Jordan algebras over F have been extensively studied and such two-dimensional algebras have been classified. In this talk (based on [1]) we classify the two-dimensional power-associative commutative algebras over F. As consequence, we obtain a classification of two-dimensional Jordan algebras over F and prove that there exists, up to isomorphism, a unique two-dimensional nonassociative Jordan algebra. The construction of this algebra can be generalized naturally to produce a Jordan algebra D with an arbitrary dimension. If F is infinite, we determine a finite basis for the polynomial identities of D, as well of all associative Jordan algebras of dimension two. This is joint work with D. Diniz (UFCG), D. Gonçalves (UFSCar) and V. Silva (UFMG).

References

[1] D. Diniz, D. Gonçalves, V. Silva e M. Souza, *Two–dimensional Jordan algebras: Their classification and polynomial identities*, LINEAR ALGEBRA AND ITS APPLICATIONS, **664**, 104–125, 2023.

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