Exploring the impact of temperature on the efficacy of replacing the wild *Aedes aegypti* population by *Wolbachia*-carrying one

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Abstract

A non-autonomous time-delayed differential system, with time-varying delay, is proposed to reproduce the competitive dynamics of *Wolbachia*-infected and non-infected mosquito populations in several scenarios that differ by daily environment temperature, the bacterial strain that promotes the infection on mosquito, and the release guidelines of infected mosquitoes. Both mosquito entomological parameters and infection's traits depend on temperature, which *per si* depends on time. Therefore, inspired by the literature on ectotherms populations, functional forms are proposed to describe the rates of birth, development, and survival (or mortality) of *Ae. aegypti* as a function of temperature, as well as the rate of *Wolbachia*-infection loss.

Mathematical model

Non-autonomous delay differential model [1, 2]:

$$\begin{aligned}
\frac{dN_u(t)}{dt} &= r_u(1-q\nu(t-\tau(t)))b_u(T(t-\tau(t)))N_u(t-\tau(t))S_u(t)\phi(T(t-\tau(t))) \\
&\quad +(1-\xi\sigma(t))r_w b_w(T(t-\tau(t)))N_w(t-\tau(t))S_w(t)\phi(T(t-\tau(t))) \\
&\quad -d_u(T(t))N_u(t)+\theta(T(t))N_w(t), \\
\frac{dN_w(t)}{dt} &= r_w\xi\sigma(t)b_w(T(t-\tau(t)))N_w(t-\tau(t))S_w(t)\phi(T(t-\tau(t))) \\
&\quad -(\theta(T(t))+d_w(T(t)))N_w(t), \\
\frac{dS_u(t)}{dt} &= S_u(t)\left[\frac{m(T(t))d_{uJ}(T(t-\tau(t)))}{m(T(t-\tau(t)))}-d_{uJ}(T(t))\right], \\
\frac{dS_w(t)}{dt} &= S_w(t)\left[\frac{m(T(t))d_{wJ}(T(t-\tau(t)))}{m(T(t-\tau(t)))}-d_{wJ}(T(t))\right], \\
\frac{d\sigma(t)}{dt} &= \sigma(t)\left[\frac{m(T(t))\theta_J(T(t-\tau(t)))}{m(T(t-\tau(t)))}-\theta_J(T(t))\right], \\
\frac{d\tau(t)}{dt} &= 1-\frac{m(T(t))}{m(T(t-\tau(t)))},
\end{aligned} \tag{1}$$

with

$$\nu(t-\tau(t)) = \frac{(1-r_w)N_w(t-\tau(t))}{\epsilon(1-r_u)N_u(t-\tau(t))+(1-r_w)N_w(t-\tau(t))}.$$

and

$$\phi(T(t-\tau(t))) = e^{-\eta(T(t-\tau(t)))(r_uN_u(t-\tau(t))+r_wN_w(t-\tau(t)))}$$

The initial condition is $N_u(t) = 200 + 200\left|\cos\left(t+\frac{\pi}{2}\right)\right|$, and $N_w(t) = 0$, for $t \in [-\tau, 0]$, $S_u(0) = e^{-\tau(0)d_{uJ}(T(0))}$, $S_w(0) = e^{-\tau(0)d_{wJ}(T(0))}$, and $\sigma(0) = e^{-\tau(0)\theta_J(T(0))}$ with $\tau(0) = m(T(0))^{-1}$.

If $T := T(t) = T(t-\tau)$, $\nu := \nu(t-\tau)$, and $r := r_u = r_w$, then System (1) admits three equilibria: (i) the equilibrium of extinction of both populations, (ii) persistence of the wild population and extinction of the infected population, (iii) and coexistence of the populations.

Silico experiments

Assuming a periodic variation of temperature over a year, the temperature function is given by

$$T(t) = T_M - \sigma_T \cos\left(\frac{2\pi t}{365}\right)$$

where T_M and σ_T are the mean temperature and its variation.

How can we measure the efficiency of the technique?

$$E_k = 1 - \frac{I_c}{I_0}, \quad \text{with} \quad I_j = \int_{t_i}^{t_f} N_u\, dt$$

where I_j with $j = \{0, c\}$ measures the cumulative number of non-infected mosquitoes observed between t_i and t_f.

The infection prevalence is measured as $P = \dfrac{N_w}{N_w + N_u}$.

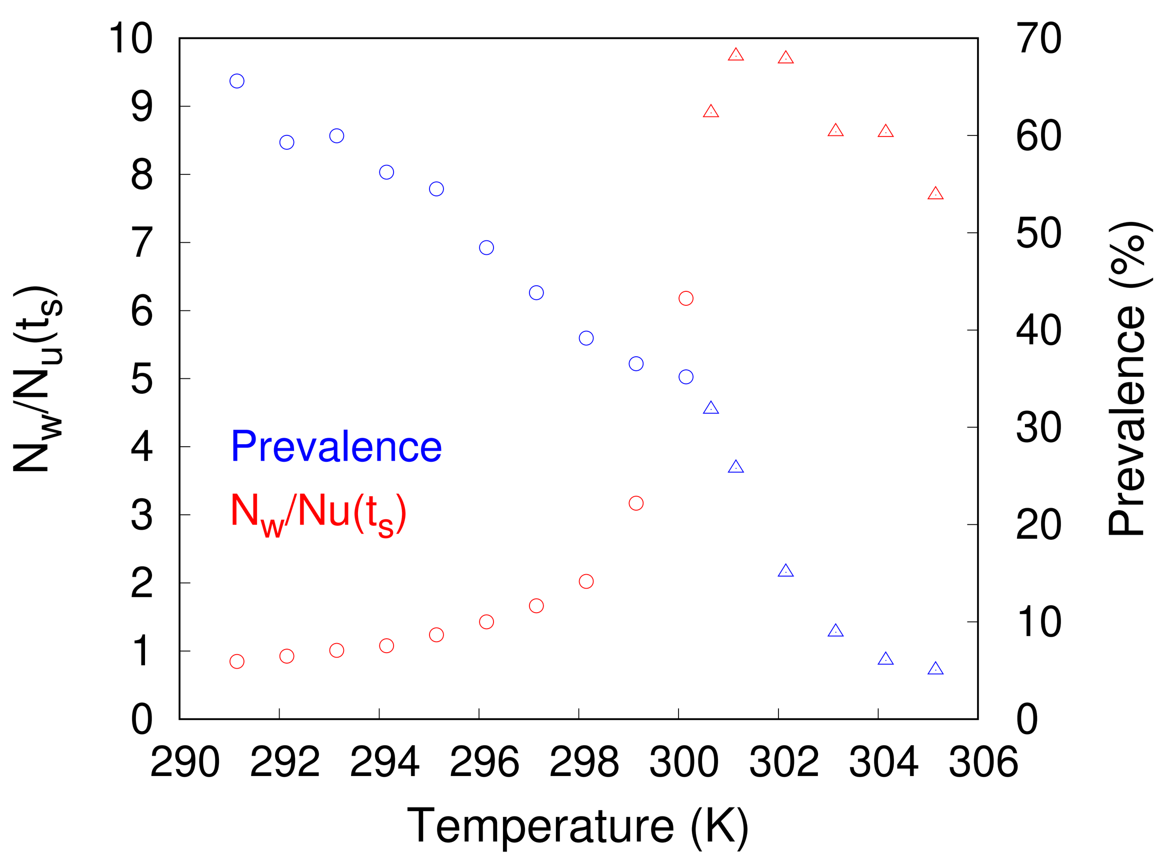

Figure 1: The ratio of infected to non-infected mosquito population (in red) and the prevalence of infection (in blue) as a function of temperature. The ratio $N_w/N_u(t_s)$ corresponds to a reduction of 50% (in ⊙), and of $< 50\%$ (in △) on N_u population. The prevalence is measured during the last thirty days of simulation.

Temperature data in Niterói - RJ.

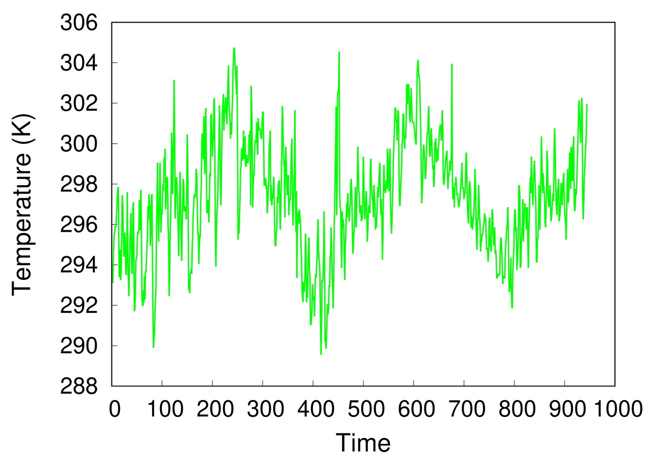

Figure 2: Temperature versus time from 2020 to 2022.

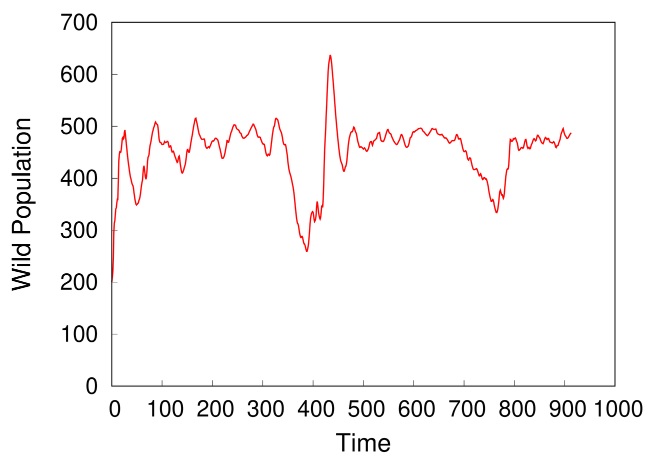

Figure 3: Non-infected population non-infected one.

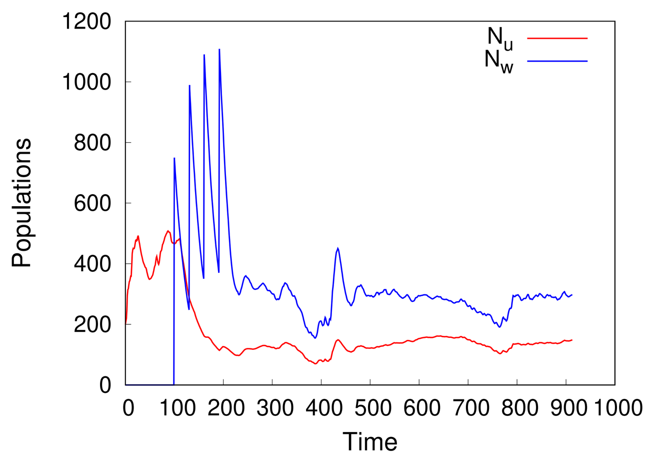

Figure 4: Non-infected and infected populations for $\xi = 0.99$ and $q = 0.95$.

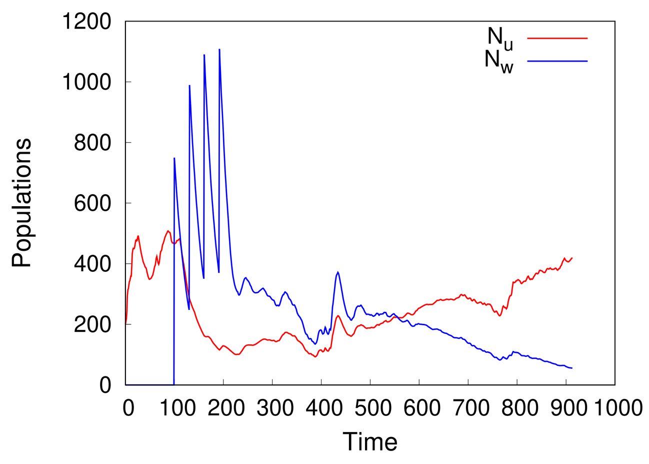

Figure 5: Non-infected and infected populations for $\xi = 0.99$ and $q = 0.8$.

Conclusions

Four releases delayed by 7 days each were more efficient than one; if one release is done, the best moment to implement it is during the favorable period when mosquito population is high; increasing the probability of cytoplasmic incompatibility and maternal inheritance can optimize the efficiency of the technique.

References

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