Foliations on rationally connected manifolds

[João Paulo Figueredo]

Universidade Federal de Minas Gerais;
joaoplf@mat.ufmg.br

Abstract:

If $\mathcal{F}$ is a regular foliation by curves on a Hirzebruch surface $f\colon S \to \mathbb{P}^1$, then one can easily see (as done in [3]) that $\mathcal{F}$ is induced by f or by a $\mathbb{P}^1$-bundle transverse to f, in which case $S = \mathbb{F}_0$. Later, Brunella shows in [1] that these are all the examples of regular foliations on rational surfaces.

It is then natural to consider the problem of classification of regular foliations on rationally connected manifolds. It is conjectured, by Touzet (see [2]), that all such foliations are algebraically integrable, with rationally connected leaves. This is confirmed in dimension two by Brunella's result. This is also confirmed for foliations on weak Fano manifolds, which are projective manifolds X having $-K_X$ nef and big, by Druel's work ([2]).

In this talk I will prove Touzet's conjecture in the case of rationally connected manifolds X having $\dim(X) = 3$ and $-K_X$ nef, and having a regular foliation with codimension one. This gives a generalization of Druel's result in the case of threefolds and codimension one foliations.

References

[1] Brunella, Marco. *Feuilletages holomorphes sur les surfaces complexes compactes.* Annales scientifiques de l'École Normale Supérieure, 4e série, 30, no. 5 (1997), 569 – 594.

[2] Druel, Stéphane. *Regular foliations on weak Fano manifolds.* Annales de la Faculté des sciences de Toulouse : Mathématiques, Ser. 6, 26, no. 1 (2017), 207 – 217.

[3] Gomez-Mont, Xavier. *Holomorphic Foliations in Ruled Surfaces.* Transactions of the American Mathematical Society 312, no. 1 (1989): 179 – 201.