Flows of geometric structures

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We develop an abstract theory of flows of geometric H-structures, i.e., flows of tensor fields defining H-reductions of the frame bundle, for a closed and connected subgroup $H \subset \mathrm{SO}(n)$, on any connected and oriented n-manifold with sufficient topology to admit such structures.

The first part of the article sets up a unifying theoretical framework for deformations of H-structures, by way of the natural infinitesimal action of $\mathrm{GL}(n, \mathbb{R})$ on tensors combined with various bundle decompositions induced by H-structures. We compute evolution equations for the intrinsic torsion under general flows of H-structures and, as applications, we obtain general Bianchi-type identities for H-structures, and, for closed manifolds, a general first variation formula for the L^2-Dirichlet energy functional $\mathcal{E}$ on the space of H-structures.

We then specialise the theory to the negative gradient flow of $\mathcal{E}$ over isometric H-structures, i.e., their harmonic flow. The core result is an almost monotonocity formula along the flow for a scale-invariant localised energy, similar to the classical formulae by Chen–Struwe for the harmonic map heat flow. This yields an ϵ-regularity theorem and an energy gap result for harmonic structures, as well as long-time existence for the flow under small initial energy, with respect to the L^∞-norm of initial torsion, in the spirit of Chen–Ding. Moreover, below a certain energy level, the absence of a torsion-free isometric H-structure in the initial homotopy class imposes the formation of finite-time singularities. These seemingly contrasting statements are illustrated by examples on flat n-tori, so long as $\pi_n(\mathrm{SO}(n)/H) \neq \{1\}$; e.g. when $n = 7$ and $H = \mathrm{G}_2$, or $n = 8$ and $H = \mathrm{Spin}(7)$

References

[1] D. Fadel, E. Loubeau, A. J. Moreno & H. N. Sá Earp, *Flows of geometric structures*, arXiv: 2211.05197 [math.DG]