Separating the edges of a graph by a linear number of paths

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A *separating path system* of a graph G is a set $\mathcal{P}$ of paths in G with the following property: for every pair (e, f) of edges in $E(G)$ there exists a path in $\mathcal{P}$ that contains e but not f. In 2022, Letzter proved that any graph of order n admits a separating path system with $O(n \log^\star n)$ paths. We improve this upper bound to $19n$, thus answering a question of Katona (2013) and confirming a conjecture independently posed by Balogh, Csaba, Martin, and Pluhár (2016) and by Falgas-Ravry, Kittipassorn, Korándi, Letzter, and Narayanan (2014). In essence, our proof uses Pósa rotation–extension to reduce the general problem to graphs that contain Hamiltonian paths. This is a joint work with Marthe Bonamy, François Dross, Tássio Naia, and Jozef Skokan.