Local central limit theorem for triangle counts in sparse random graphs

Letícia Mattos (University of Illinois Urbana-Champaign)

Let X_H be the number of copies of a fixed graph H in $G(n,p)$. In 2016, Gilmer and Kopparty conjectured that a local central limit theorem should hold for X_H as long as H is connected, $p \gg n^{-1/m(H)}$ and $n^2(1-p) \gg 1$, where $m(H)$ denotes the m-density of H. Recently, Sah and Sahwney showed that the Gilmer–Kopparty conjecture holds for constant p. In this talk, we show that the Gilmer–Kopparty conjecture holds for triangle counts in the sparse range. More precisely, if $Cn^{-1/2} \leq p \leq 1/2$, for some large constant $C > 0$, then

$$\sup_{x \in \mathcal{L}} \left| \frac{1}{\sqrt{2\pi}} e^{-x^2/2} - \sigma \cdot \mathbb{P}(X^* = x) \right| \to 0,$$

where $\sigma = \mathbb{V}\mathrm{ar}(X_{K_3})$, $X^* = (X_{K_3} - \mathbb{E}(X_{K_3}))/\sigma$ and $\mathcal{L}$ is the support of X^*. By combing our result with the results of Röllin–Ross and Gilmer–Kopparty, this establishes the Gilmer–Kopparty conjecture for triangle counts for $n^{-1} \ll p < c$, for any constant $c \in (0,1)$. This is the first local central limit theorem for subgraph counts above the m_2-density.