Area-minimizing integral currents: singularities and structure

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Area-minimizing integral currents were introduced by De Giorgi, Federer, and Fleming to build a successfull existence theory for the *oriented* Plateau problem. While celebrated examples of singular minimizers were discovered soon after, a first theorem which summarizes the work of several mathematicians in the 60es and 70es (De Giorgi, Fleming, Almgren, Simons, and Federer) and a second theorem of Almgren from 1980 give general dimension bounds for the singular set which match the one of the examples, in codimension 1 and in general codimension respectively.

In joint works with Anna Skorobogatova and Paul Minter we prove that in higher codimension the singular set is (m-2)-rectifiable and the tangent cone is unique at \mathcal{H}^{m-2} a.e. point. Independently and at the same time, a proof of the same result has been discovered also by Krummel and Wickramasekera. This theorem is the counterpart, in general codimension, of a celebrated work of Leon Simon in the nineties for the codimension 1 case. Moreover, a recent theorem by Liu proves that the singular set can in fact be a fractal of any Hausdorff dimension $\alpha \leq m-2$, indicating that the above structure theorem is indeed close to optimal.