

# Quasi stationary distributions in large population birth and death processes, resilience and spectral gap

Pierre Collet<sup>1</sup>

<sup>1</sup> Centre de Physique Théorique, Institut Polytechnique de Paris

We consider one or multi species birth and death processes whose population size is controlled by a large parameter  $K$ . We assume that the dynamical system obtained by taking the rescaled infinite parameter limit has a unique globally stable fixed point. Under suitable technical assumptions we show the existence and uniqueness of a quasi stationary distribution (qsd) when the evolution avoids the disappearance of the whole population. For large population this qsd describes the statistics of the system on a time scale exponentially large in the parameter  $K$  while the time scale of convergence to this distribution is at most of the order  $\log(K)$ .

Time correlations of the microscopic fluctuations can be used to estimate the spectrum of the Jacobian of the dynamical system at the fixed point (macroscopic resilience).

We also show that for one specie, although the time scale of contraction to the qsd has an upper bound of order  $\log(K)$  the spectral gap in a suitable Hilbert space is of order one. We completely describe the asymptotic of the spectrum and discuss the relation between the spectral gap and the resilience.