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GENERAL COHERENT SYSTEMS

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Maps from a curve X to projective spaces are studied in terms of linear systems, i.e., pairs (L,V), consisting of a line bundle L on X and a linear subspace $V \subset H^0(X,L)$. These pairs were studied within classical Brill–Noether theory. In order to investigate, e.g., moduli spaces of semistable vector bundles on X, one looks at higher rank *Brill–Noether pairs* or *coherent systems*. These are pairs (E,V) composed of a vector bundle E on X and a linear subspace $V \subset H^0(X,E)$. It is natural to fix the rank r and the degree d of E as well as the dimension s of V. Unlike the case of line bundles, i.e., $r=1$, it is necessary to introduce a stability condition and restrict to the corresponding semistable coherent systems in order to obtain reasonable moduli spaces for coherent systems of type (r,d,s). Brill–Noether theory for higher rank vector bundles turned out to be substantially different from Brill–Noether theory for line bundles, and the basic questions have not yet been settled in full generality.

Now, let us fix a reductive group G, a finite dimensional vector space V, and a representation $\varrho\colon G \longrightarrow \mathrm{GL}(V)$ of G on V. Then, we may associate with any principal G-bundle P on X a vector bundle $\mathcal{P}_\varrho$ on X with typical fiber V. Let us also fix a line bundle L on X. Then, a general coherent system consists of a principal G-bundle $\mathcal{P}$ on X and a subspace $V \subset H^0(X,\mathcal{P}_\varrho \otimes L)$. For $G=\mathrm{GL}(r)$, $\varrho = \mathrm{Ad}_G$ the adjoint representation, and $L=\omega_X$ the canonical line bundle, the study of these objects has been proposed by Brambila-Paz, García-Prada, and Gothen. In this case, it is expected that the resulting moduli spaces will have applications to the geometry of moduli spaces of Higgs bundles. Another interesting case arises for $G=\mathrm{GL}(r)$, ϱ a symmetric power of the standard representation, and L an arbitrary line bundle. The resulting objects are linear systems of divisors in projective bundles over X.

In the talk, I will propose a notion of semistability for general coherent systems and present results on the existence of moduli spaces for $G=\mathrm{GL}(r)$, ϱ a tensor power of the standard representation, and L the trivial line bundle.[1]

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[1] This case does not include coherent Higgs systems which are being studied by Edgar Castañeda, a PhD student of Leticia Brambila-Paz.