Artin-Schreier extensions of minimal genus

Herivelto Borges
(Joint work with N. Arakelian and M. Coutinho)

Instituto de Ciências Matemáticas e de Computação
Universidade de São Paulo

Abstract

Let $\mathcal{X}$ be a nonsingular, irreducible, projective curve of genus $g(\mathcal{X})$, defined over an algebraically closed field $\mathbb{K}$ of characteristic $p > 0$, and let $\mathbb{K}(\mathcal{X})$ be its function field. If $\mathcal{X}$ has zero p-rank, then any degree-p Galois extension of $\mathbb{K}(\mathcal{X})$ is ramified, and thus

$$g(\mathcal{Y}) > (g(\mathcal{X}) - 1)p + 1, \tag{1}$$

where $\mathcal{Y}$ is the curve corresponding to an Artin-Schreier extension $\mathbb{K}(\mathcal{Y})$ of $\mathbb{K}(\mathcal{X})$. Bearing in mind the above inequality, a natural problem is that of characterizing the Artin-Schreier extensions of $\mathbb{K}(\mathcal{X})$ with smallest possible genus.

In this talk, we discuss the problem of characterizing such Artin-Schreier extensions of certain supersingular curves. Our focus for the base curves of such extensions will be with curves with large automorphism groups, such as the Hermitian curve. For this particular curve, we will show that such a p-extension is, up to isomorphism, a unique curve with very special arithmetic and geometric properties. For instance, we will see that one can easily compute its Zeta function and automorphism group, and show that the latter is a relatively large group whose p-Sylow subgroups attain the Nakajima bound for curves of zero p-rank.