

Holomorphic correspondences, matings, and Klein's combination theorem

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Nearly three decades ago, Chris Penrose and I introduced a family of quadratic holomorphic correspondences F_a which we proved in certain cases realise matings between quadratic maps and the modular group. In our 1994 article in *Inventiones* we conjectured that for all parameter values in the connectedness locus M_Γ of this family the correspondence F_a is such a mating, and that M_Γ is homeomorphic to the classical Mandelbrot set M . By applying the theory of parabolic-like mappings introduced by Luna Lomonaco in her 2012 thesis, Lomonaco and I succeeded in proving both conjectures.

I will give an overview of some of the ideas involved in constructing holomorphic matings and proving results about them, at the same time highlighting new questions in what is still a largely unexplored area of mathematics. Topics will include some or all of the following: the role of Klein's combination theorem; generalisation of the family F_a to higher degrees; the relationship between F_a and the (anti-holomorphic) Schwarz reflection maps of Lee, Lyubich, Makarov and Mukherjee; the application of Lomonaco's parabolic surgery to the family F_a which (thanks also to a theorem of Petersen and Roesch) culminated in our proof that M_Γ is homeomorphic to M ; and our joint work in progress on the discreteness locus K of the family F_a , the natural tessellation of $K \setminus M_\Gamma$, and the conjectured image of the 'Douady-Hubbard map' from $K \setminus M_\Gamma$ to the hyperbolic plane.