

Brjuno functions, Hölder continuity and modular forms

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In 1988 Yoccoz proved that the size of the stability domain (Siegel disk) around an irrationally indifferent fixed point in the complex plane is given by a purely arithmetic function—called Brjuno’s function—up to a more regular L^∞ correction. The Hölder interpolation conjecture (aka Marmi-Moussa-Yoccoz conjecture) states that for quadratic polynomials this correction is in fact $1/2$ -Hölder continuous. An analogous version of the conjecture stands also for other dynamical systems, including the standard family. Hölder continuity seems to be the relevant regularity for these problems also since it measures the difference between formulations of the arithmetical function corresponding to different continued fraction algorithms (Gauss, nearest integer, by-excess, ...) Surprisingly finally, very similar functions are used to study the convergence of trigonometric sums involving the divisor function, as discovered by Wilton almost a century ago, and also for the study of the differentiability properties of integrals of modular forms. The talk will be based on recent work in collaboration with Seul Bee Lee, Izabela Petrykiewicz and Tanja Schindler.