

Constructing nonhyperbolic ergodic measures (with positive entropy)

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We aim to describe the “lack” of hyperbolicity for diffeomorphisms, following the well established path considering gradually simplified systems: partially hyperbolic diffeomorphisms with one-dimensional center, skew products, step skew products with one-dimensional fiber maps, and step skew products induced by $SL(2, \mathbb{R})$ matrix cocycles. We considered the latter two cases.

Our main assumption is the existence of contracting/expanding regions (blending regions) which are mingled by the dynamics (transitivity). This guarantees the existence of nonhyperbolic measures (zero fiber Lyapunov exponent) with positive entropy. We construct nonhyperbolic ergodic measures with as large as possible metric entropy. Our approach is inspired by the concatenating periodic orbits technique of Gorodetski, Ilyashenko, Kleptsyn, and Nalski. However, this technique always results in measures with zero entropy and we aim for measures with large entropy.

I will explain how to modify this technique to produce measures with positive entropy and entropy as large as possible. In this modification, periodic orbits are replaced by horseshoes. This naive replacement carries several difficulties (as the non-uniform convergence of Birkhoff means) and opens new doors and perspectives.

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