

# Renormalization, fractal geometry, and the Newhouse phenomenon

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As discovered by Poincaré in the end of the 19th century, even small perturbations of very regular dynamical systems may display chaotic features, due to complicated interactions near a homoclinic point. In the 1960s, Smale attempted to understand such dynamics in terms of a stable model, the horseshoe, but this was too optimistic. Indeed, Newhouse showed that even in only two dimensions, a homoclinic bifurcation gives rise to particular wild dynamics, such as the generic presence of infinitely many attractors. This Newhouse phenomenon is associated to a renormalization mechanism, but also with particular geometric properties of some fractal sets within a Smale horseshoe. When considering two-dimensional complex dynamics those fractal sets become much more beautiful but unfortunately also more difficult to handle.