Speaker: Henryk Żołądek (University of Warsaw)

Title: Normal forms of planar vector fields

Abstract: Recall that a planar vector field singularity is elementary if at least one of eigenvalues of its linearization is nonzero. The formal normal forms of elementary singularities were found long ago. Also the analytic properties of these forms are generally known (the small divisors and the Stokes phenomena).

But the non-elementary singularities have turned out resistant. We know the Takes normal form for the Bogdanem–Takes singularity, which has turned out analytic. But it is only a paranormal form, it is not unique. Investigators tried to approach this problem using splitting of the space of vector fields into Hamiltonian ones and parallel to some quasi-homogeneous Euler vector field.

In recent years I and my student Ewa Stróżyna have found an effective tool to treat this problem. We use another splitting.

Assume that the considered vector fields are of the form $V = V_0 + \dots$, where V_0 is a quasi-homogeneous vector field (with respect to some grading). We divide the perturbations W, as well as the vector fields Z used in the normalization, into parallel to V_0 and transversal to V_0. The first ones are of the form fV_0, i.e., are defined by one function f, and the transversality of Z is measured by another function $g = Z \wedge V_0/\partial_x \wedge \partial_y$. The standard homological operator $Z \longmapsto [V_0, Z]$ is split into two differential operators $C\,(V_0)$ and $D\,(V_0)$ acting on f and g respectively. It turns out that $\ker C\,(V_0)$ consists of first integrals of V_0 and $\ker D\,(V_0)$ consists of inverse integrating multipliers for V_0. Moreover, the cokernels of these operators are defined via periods of suitable Schwarz–Christoffel functions.

Using this approach we have completed the list of normal forms for the Bogdanem–Takes singularity and in the case when V_0 is homogeneous quadratic. Moreover, in the Bogdanem–Takes case we proved (non-)analyticity properties of the obtained normal forms; in the non-analytic cases we used a method developed by Ilyashenko in the small divisors problem for the elementary singularities.