Commuting probability for subgroups of a finite group

Pavel Shumyatsky

This is a joint work with Eloisa Detomi (University of Padova).

If K is a subgroup of a finite group G, the probability that an element of G commutes with an element of K is denoted by $Pr(K,G)$. The probability that two randomly chosen elements of G commute is denoted by $Pr(G)$. A well known theorem, due to P. M. Neumann, says that if G is a finite group such that $Pr(G) \geq \epsilon$, then G has a nilpotent normal subgroup T of class at most 2 such that both the index $[G:T]$ and the order $|[T,T]|$ are ϵ-bounded.

In the talk we will discuss a stronger version of Neumann's theorem: if K is a subgroup of G such that $Pr(K,G) \geq \epsilon$, then there is a normal subgroup $T \leq G$ and a subgroup $B \leq K$ such that the indexes $[G:T]$ and $[K:B]$ and the order of the commutator subgroup $[T,B]$ are ϵ-bounded.

We will also discuss a number of corollaries of this result. A typical application is that if in the above theorem K is the generalized Fitting subgroup $F^*(G)$, then G has a class-2-nilpotent normal subgroup R such that both the index $[G:R]$ and the order of the commutator subgroup $[R,R]$ are ϵ-bounded.

Department of Mathematics, University of Brasilia, Brasilia-DF, 70910-900 Brazil

E-mail address: pavel2040@gmail.com