Smooth vs symbolic dynamics of compact manifold

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For a finite alphabet A, let A^{ω} be the set of all infinite words over A. A group Γ is called freely self-similar if it admits a free self-similar action on A^{ω} for some alphabet A. We investigate the following conjecture:

Conjecture: Let M be a smooth compact manifold. If M is endowed with an Anosov diffeomorphism, then its fundamental group is freely self-similar.

Theorem: the conjecture holds if M is a nilmanifold

Indeed the proof is based on a theorem about rational points of algebraic tori. As a consequence, our conjecture is a consequence of the classical Anosov-Smale conjecture.