The multiplicative random walk

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A random multiplicative function f is defined as follows: On the prime numbers p, the values $f(p)$ are given by an iid sequence of Bernoulli taking ± 1 with half probability each, and on the other positive (squarefree) integers n, $f(n)$ is defined accordingly the prime factorization of n. For instance, since $30 = 2 \times 3 \times 5$, we have that $f(30) = f(2)f(3)f(5)$. Thus, the randomness is only at the primes. This has been introduced by Wintner in the 40's to serve as a probabilistic model for the Möbius function, a number-theoretic function which encodes the Riemann hypothesis. A natural question is if the multiplicative random walk given by the partial sums of a random multiplicative function f is recurrent. In this talk I will explain a recent work jointly with Winston Heap and Jing Zhao on this topic.