Free boundaries in segregation problems

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Abstract

We first consider classes of variational problems for densities that repel each other at distance. Examples are given by the minimizers of Dirichlet functional or the Rayleigh quotient

$$D(\mathbf{u}) = \sum_{i=1}^{k} \int_\Omega |\nabla u_i|^2 \quad \text{or} \quad R(\mathbf{u}) = \sum_{i=1}^{k} \frac{\int_\Omega |\nabla u_i|^2}{\int_\Omega u_i^2}$$

over the class of $H^1(\Omega, \mathbb{R}^k)$ functions attaining some boundary conditions on $\partial\Omega$, and subjected to the constraint

$$\mathrm{dist}(\{u_i > 0\}, \{u_j > 0\}) \geq 1 \qquad \forall i \neq j.$$

As second class of problems, we consider energy minimizers of Dirichlet energies with different metrics

$$D(\mathbf{u}) = \sum_{i=1}^{k} \int_\Omega \langle A_i \nabla u_i, \nabla u_i \rangle$$

with constraint

$$u_i(x) \cdot u_j(x) = 0, \qquad \forall x \in \Omega \; , \forall i \neq j.$$

For these problems, we investigate the optimal regularity of the solutions, prove a free-boundary extremality condition, and derive some preliminary results characterising the emerging free boundary.

References

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[2] N. Soave and S. Terracini, *An anisotropic monotoncity formula, with applications to some segregation problems*, preprint (2020), (http://arxiv.org/abs/2004.08853)