Time-periodic Gelfand-Shilov spaces and global hypoellipticity for a class of evolution equations

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In this talk we present a class of *time-periodic Gelfand-Shilov* spaces, namely, the complex-valued functions u defined on $\mathbb{T} \times \mathbb{R}^n$ satisfying the following: there exist $R, C > 0$ such that

$$\sup_{t \in \mathbb{T}, x \in \mathbb{R}^n} |x^\alpha \partial_x^\beta \partial_t^j u(t,x)| \leq R C^{|\alpha+\beta|+j} (j!)^\sigma (\alpha! \beta!)^\mu,$$

for every $\alpha, \beta \in \mathbb{N}_0^n, j \in \mathbb{N}_0$, where $\mathbb{T} \simeq \mathbb{R}/2\pi\mathbb{Z}$.

We develop a Fourier analysis for these spaces in view of the eigenfunction expansions on $\mathbb{R}^n$ given by an operator

$$P = \sum_{|\alpha|+|\beta| \leq m} c_{\alpha,\beta} x^\beta D_x^\alpha, \ D_x^\alpha = (-i)^{|\alpha|} \partial_x^\alpha,$$

such that $P^* P = P P^*$ and

$$p_m(x,\xi) = \sum_{|\alpha|+|\beta|=m} c_{\alpha,\beta} x^\beta \xi^\alpha \neq 0, \quad (x,\xi) \neq (0,0).$$

As an application, we study the global hypoellipticity of the operator

$$L = D_t + c(t) P, \ D_t = i^{-1} \partial_t,$$

where $c = c(t)$ belongs to some Gevrey class on $\mathbb{T}$.