Multipliers and Convolutors of the space $\mathcal{S}_\omega(\mathbb{R}^N)$ of the ω-ultradifferentiable rapidly decreasing functions of Beurling type

Angela A. Albanese[1], Claudio Mele[2],

[1] University of Salento

[2] University of Salento

In the last years the attention has focused on the space $\mathcal{S}_\omega(\mathbb{R}^N)$ of the ultradifferentiable rapidly decreasing functions of Beurling type, as defined by Björck. Motivated by this line of research, we introduce and study the space $\mathcal{O}_{M,\omega}(\mathbb{R}^N)$ of the slowly increasing functions of Beurling type in the setting of ultradifferentiable function space in the sense of Braun, Meise and Taylor. We show that $\mathcal{O}_{M,\omega}(\mathbb{R}^N)$ is the space of the multipliers of the space $\mathcal{S}_\omega(\mathbb{R}^N)$ and of its dual space $\mathcal{S}'_\omega(\mathbb{R}^N)$. We define and compare some locally convex topologies of which $\mathcal{O}_{M,\omega}(\mathbb{R}^N)$ can be naturally endowed. We also introduce the space $\mathcal{O}_{C,\omega}(\mathbb{R}^N)$ of the very slowly increasing functions of Beurling type and we prove that its strong dual $\mathcal{O}'_{C,\omega}(\mathbb{R}^N)$ is the space of convolutors of the space $\mathcal{S}_\omega(\mathbb{R}^N)$ and of its dual space $\mathcal{S}'_\omega(\mathbb{R}^N)$. We establish that the Fourier transform is an isomorphism from $\mathcal{O}'_{C,\omega}(\mathbb{R}^N)$ onto $\mathcal{O}_{M,\omega}(\mathbb{R}^N)$. This isomorphim is topological when the former space is endowed with the strong operator lc-topology induced by $\mathcal{L}_b(\mathcal{S}_\omega(\mathbb{R}^N))$ and the last space is endowed with its natural lc-topology.