

Upper bounds for rational points

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The question “what are the solutions in rational numbers to an algebraic equation?” is the one that drives the subject of Diophantine geometry, and has for centuries. It is much, much too hard. So instead one might ask: “how many solutions does an algebraic equation have?” Still too hard. One might thus be willing to settle for “Are there good upper bounds for the number of solutions an algebraic equation have?” and here at last there are some good general results. I’ll talk about what is known, with special attention to the distinction between uniform results (those with no dependence, or minimal dependence, on the particular equation at issue) and non-uniform results (which depend strongly on the arithmetic properties of the individual equation), and will close with a new result (joint with Brian Lawrence and Akshay Venkatesh) showing that there are in a sense “very few” hypersurfaces in projective space whose determinant takes a fixed integer value — a non-uniform bound which uses in a critical way the existence of uniform bounds developed in the last twenty years.